Assignment 1
Semantics, WS 2013/14

Prof. Dr. Gert Smolka, Steven Schäfer
www.ps.uni-saarland.de/courses/sem-ws13/

Exercise 1.1 We consider the Coq embedding of Church numerals.

```
Definition Nat : Prop := forall X : Prop, X -> (X -> X) -> X.

Definition zero : Nat := fun X x f => x.
Definition succ : Nat -> Nat := fun n X x f => f (n X x f).

Definition N (n : nat) : Nat := nat_iter n succ zero.
```

Define functions for addition, multiplication and exponentiation for *Nat* and show their correctness in Coq. For instance:

```
Definition add : Nat -> Nat -> Nat := ...

Lemma N_add m n : N (m + n) = add (N m) (N n).
Proof...
```

Exercise 1.2 Find a λ-term ω_f such that $\omega_f\omega_f \succ f(\omega_f\omega_f)$, where f is a free variable in ω_f.

Exercise 1.3 (Challenge) Church's original representation of the natural numbers swapped the argument order.

$$\hat{n} = \lambda f\, x.f^n x$$

A nice feature of this representation is that there is a remarkably simple way of defining exponentiation. Try to find it and prove its correctness.