Assignment 3
Semantics, WS 2013/14

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Exercise 3.1 We define the Scott numeral for $n \in \mathbb{N}$ by recursion:

$$\begin{aligned}\overline{0} &= \lambda x f.x\\ \overline{Sn} &= \lambda x f.f\overline{n}\end{aligned}$$

Define addition and multiplication on Scott numerals. Show the correctness of your definition. You may assume the existence of a fixed point combinator Y satisfying the equivalence $Ys \equiv s(Ys)$ for all terms s.

Exercise 3.2 Define the Scott and Church representations of lists. Define functions to compute the length of a list for both representations.

Exercise 3.3 Show that there is no normal fixed point combinator. More specifically, let R be a closed term satisfying the equivalence $Rx \equiv x(Rx)$. Prove that R does not have a normal form.

Exercise 3.4 SK-terms are given by the following grammar:

$$s, t ::= S \mid K \mid s\,t$$

We define the following reduction relation on SK-terms:

$$\begin{aligned}K\ x\ y &\to x\\ S\ f\ g\ x &\to f\ x\ (g\ x)\end{aligned}$$

Find SK-terms I, C, B, Y with the following reduction behaviour:

$$\begin{aligned}I\ x &\to^* x\\ C\ x\ y &\to^* y\ x\\ B\ f\ g\ x &\to^* f\ (g\ x)\\ Y\ f &\to^* f\ (Y\ f)\end{aligned}$$