



## Assignment 6 Semantics, WS 2013/14

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Read in the lecture notes: Chapter 2.5 - 2.8

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**Exercise 6.1** Let  $R$  be a relation on  $X$ .

- Show that for normal  $x$  we have  $SN R x$ .
- Prove that  $SN R \leftrightarrow SN R^+$ , where  $R^+$  is the transitive closure of  $R$ .
- Use this to show the strong induction principle for  $SN$ .
- Show the preimage lemma.
- Prove that if  $SN R x$ , then there is no infinite path from  $x$ .

**Exercise 6.2** Show that  $>$  is a terminating relation on  $\mathbb{N}$ .

**Exercise 6.3** A relation  $R$  is called “classical” if we can decide whether  $x$  is reducible or normal according to  $R$ .

- Show that for classical relations,  $SN R \rightarrow WN R$ .
- Now show that the requirement is necessary. Assuming  $SN R \rightarrow WN R$  for every relation  $R$ , derive the law of excluded middle.

**Exercise 6.4** Let  $\rho$  be a reduction operator satisfying the triangle property for  $R$ . Prove the following.

- diamond*  $R$
- reflexive*  $R \rightarrow \text{sound } R \rho$
- $Rxy \rightarrow R(\rho x)(\rho y)$
- $Rxy \rightarrow R(\rho^n x)(\rho^n y)$
- cofinal*  $R \rho$

**Exercise 6.5** Let  $\rho$  be a reduction operator that is sound for  $R$ . Prove the following.

- normal*  $R x \rightarrow \rho x = x$
- cofinal*  $R \rho \rightarrow (x \Downarrow^R y \leftrightarrow \exists n. y = \rho^n x \wedge \text{normal } R y)$

**Exercise 6.6** Show Newman’s Lemma: For terminating relations  $R$ , local confluence implies confluence.

**Exercise 6.7** Find a property of predicates which is not stable under infinite intersection.

**Exercise 6.8** Find a relation  $R$  along with a sound and cofinal reduction strategy  $\rho$  such that  $\rho x = x$  does not imply *normal*  $R x$ .